Institution: Department of Mathematics

Name: Ichikawa Takashi

Theme: Arithmetic geometry and mathematical physics related with string theory

I am mainly interested in studying the relationship between arithmetic geometry and mathematical physics, and up to the present I have studied mainly arithmetic geometry on algebraic curves (Riemann surfaces in analytic geometry) and their moduli space, which are main objects of the investigation in string theory, and its application to mathematical physics related with string theory. In the following, I will give a summary of my results on this theme. First, I obtained fundamental results on automorphic forms (Teichmueller modular forms) on the moduli space of algebraic curves by using Schottky and Mumford uniformization theories of algebraic curves. In the proof of these results, universal periods of algebraic curves discovered by me play an important role.

Second, I constructed arithmetic uniformization theory of algebraic curves (higher genus version of Tate curves) unifying Schottky and Mumford uniformization theories, and using this I obtained results on Teichmueller modular forms over the ring of integers: finitely generatedness and explcit description in low genus cases of the ring of Teichmueller modular forms, and a solution to the Schottky problem. In this study, I also proved that Teichmueller modular forms appearing in string theory express the "difference" between the ring of Siegel modular forms and that of Teichmueller modular forms.

Furthermore, using the above universal periods, I showed that there exists a universal power series expression of quasi-periodic (theta function) solutions to systems of soliton equations, and that by specializing parameters to p-adic numbers, the universal solution gives rise to solutions to the systems of soliton equations (these results are extended to the case when Riemann surfaces have infinite genus). Recently, using the above arithmetic uniformization theory of algebraic curves, I constructed arithmetic Teichmueller groupoids whose existence was conjectured by Grothedieck, and also I described the Galois action on the groupoids and the monodromy representation (associated with conformal field theory by Tsuchiya-Ueno-Yamada) of the groupoids.